The Effect of Eccentricity on the Terminal Velocity of the Cylinder in a Falling Cylinder Viscometer: Experimental Verification for Newtonian Fluids

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Chen, et al. [1] solved the theoretical equations describing the performance of the falling cylinder viscometer with eccentricity in Newtonian fluids. This paper presents data which verify those theoretical results.

EXPERIMENTAL PROCEDURE

The viscometer apparatus, described in detail elsewhere [2], consisted of a precision-bore Pyrex fall tube in which solid aluminum falling cylinders could be dropped through Newtonian fluids. This apparatus was immersed in a thermostatted oil bath to maintain the temperature constant at 25°C.

Each falling cylinder used in the experiments was vertically aligned within the fall tube by two sets of three steel pins, one set at the top and one at the bottom of the cylinder with the pins radiating from stems to reduce entrance and exist flow effects (see Figure 1). The pins of each set were spaced radially at intervals of 120 deg. so that the pins at the top and bottom of the cylinder were in the same vertical plane. To induce predetermined values of eccentricity, the top and bottom pins on one side of the cylinder were shortened while the other two pins at top and bottom were lengthened.

Four cylinders, with κ's of about 0.84, 0.87, 0.90, and 0.93, were used. The terminal velocity of fall for each cylinder was measured three times in each of four National Bureau of Standards oils, with viscosities ranging from 0.77 to 9.2 poise at 25°C., first with the pins at each end of equal length (to establish the concentric terminal velocity) and then for two different induced values of eccentricity created as described above. The data obtained are compared with the theoretical results of Chen, et al. [1] in Figure 2. The reproducibility of the data for one cylinder at one value of ϕ in the four oils was within $\pm$ 1%.

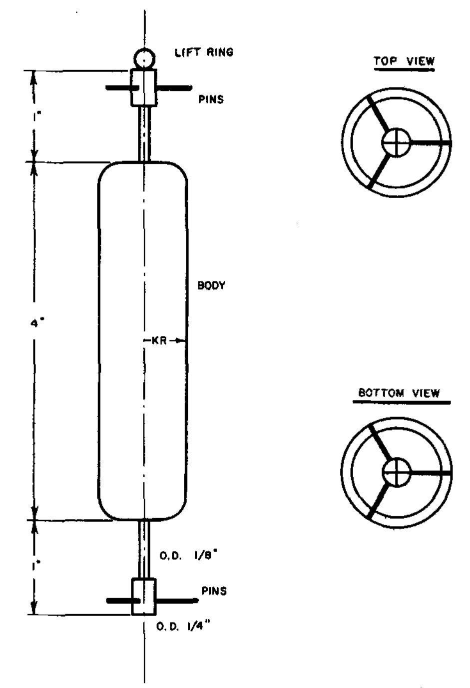

Fig. 1. Drawing of a viscometer body.

DISCUSSION

The data show good agreement with theory. That the data give lower terminal velocity ratios than predicted is to be expected since the eccentricity ratios, at which the data are plotted in Figure 2, were computed for zero tolerance between the pins and the fall tube. A small machine tolerance between pins and tube was allowed to enable the cylinder to fall freely in the fall tube. Thus the true eccentricity ratio is somewhat less than that computed for each of the cylinders. In addition, the concentric configuration for each cylinder was probably not quite concentric. The slightest amount of eccentricity would make the measured terminal velocity for the concentric case larger than it should be. Corrections made to compensate for these two factors would shift the data toward the theoretical results. Without any corrections, the error ranged from 0.2 to 4.7% with an average of 2.5%.

The slight dependence of the terminal velocity of κ

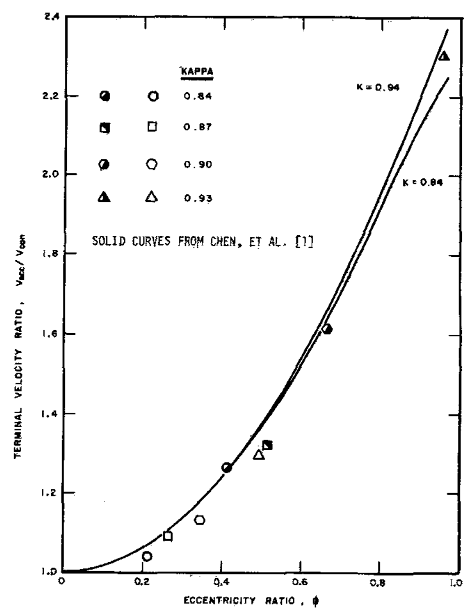

Fig. 2. Terminal velocity vs. corrected eccentricity ratio for four falling cylindrical bodies.

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could not be confirmed or denied since its effect was, in general, less than the experimental error. The only exception was at the highest eccentricity ratio investigated, $\phi = 0.96$, $\kappa = 0.93$, where the one experimental data point obtained verified the predicted dependence of terminal velocity ratio on κ.

CONCLUSION

All factors considered, the experimental data reported here provide verification of the theory developed by Chen, et al. [1], agreeing with an average deviation of 2.5%.

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NOTATION

R = inner radius of fall tube, cm.
V_{con} = terminal velocity of cylinder when concentric to fall tube, cm./sec.
V_{ecc} = terminal velocity of cylinder when eccentric to fall tube, cm./sec.
ϵ = eccentricity or displacement of axis of cylinder to axis of fall tube, cm.
κ = ratio of cylinder radius to inner radius of fall tube, dimensionless
ϕ = eccentricity ratio, $\epsilon/[R(1-\kappa)]$, dimensionless

LITERATURE CITED

1. Chen, M. C. S., J. A. Lescarboura, and G. W. Swift, *AIChE J.*, 14, 123 (1968).
2. Lescarboura, J. A., Ph.D. thesis, Univ. Kansas, Lawrence (1967).

Thermal Instability of a Horizontal Layer of Water Near 4°C.

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When a layer of fluid is subject to an adverse temperature gradient, the system is potentially unstable. The onset of convection is indicated when the Rayleigh number exceeds its critical value. Rayleigh number is conventionally defined as

$$N_{Ra} = \frac{g\alpha(\Delta T)d^3}{\nu\kappa}$$

The critical Rayleigh number is dependent upon the boundary conditions. For the case of rigid-rigid surfaces, $(N_{Ra})_{cr}$ is found to be approximately 1,700.

The problem becomes somewhat involved if the liquid possesses a maximum density (or minimum) within the lower and upper surface temperature. A good example is that of a water layer which is subject to temperatures on either side of 4°C. The density of water increases upward from the lower surface until it reaches a maximum then it decreases. In other words, part of the liquid layer is potentially unstable while the other part is stable. Stability analysis of this problem with rigid-rigid surface has been carried out by a number of investigators (2, 3) by using the following density-temperature relationship,

$$\rho - \rho_{max} = -\rho_{max}\,\gamma(T - T_{max})^2 \qquad (1)$$

A modified Rayleigh number can be defined as

$$N_{Ra} = \frac{(2\gamma\; A\Delta T)\; g(\Delta T)d^3}{\nu\kappa} \qquad (2)$$

The critical Rayleigh number is found to be a function of parameter A which is given as

$$A = \frac{T_l - T_{max}}{T_l - T_u} \qquad (3)$$

Numerical values of $(N_{Ra})_{cr}$ vs. A are shown in Figure 1. For $A < 0.25$, the asymptotic expression of Chandrasekhar can be used

$$(N_{Ra})_{cr} \sim 1186.4\left(\frac{1}{A}\right)^4 \qquad (4)$$

It is interesting to note that Rayleigh number defined by Equation (2) is always positive because of the term of $(\Delta T)^2$. Consequently, the onset of convection is possible for both heating and cooling. Furthermore, one can show that this criterion is also consistent with physical argument. For the limiting case of $A = 1$, $T_u = T_{max}$, and there is no density inversion within the liquid layer. The critical Rayleigh number becomes

$$(N_{Ra})_{cr} = \frac{g[\gamma 2(T_l - T_{max})]\,[T_l - T_{max}]}{\nu\kappa}\,d^3 = 3{,}390$$

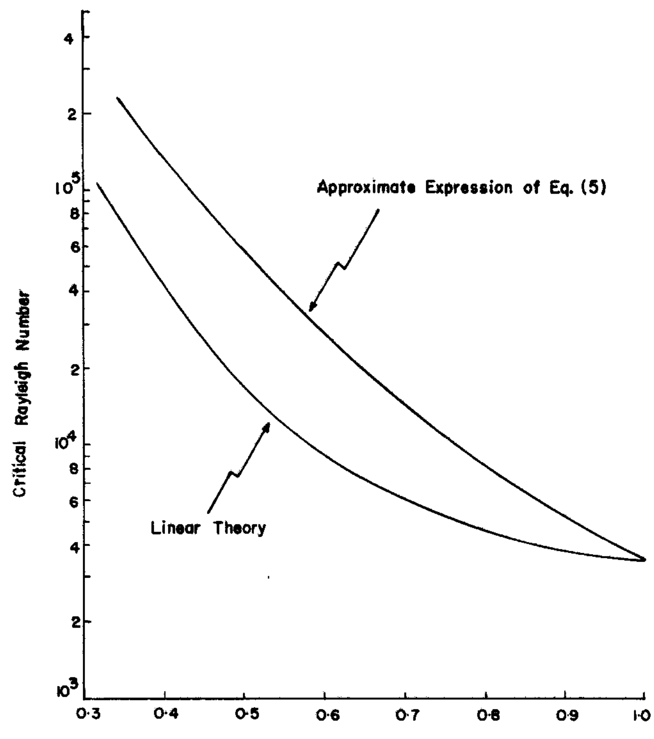

Fig. 1. Critical Rayleigh number as a function of A.